JORNADAS CHILENAS DE TEORÍA FUZZY 2025

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Jornadas Chilenas de Teoría Fuzzy 2025

JCTF2025

La realización de las Jornadas Chilenas de Teoría Fuzzy surge de la necesidad de articular, visibilizar y fortalecer la comunidad nacional de investigadores que desarrollan esta disciplina en Chile. Este encuentro busca reunir a especialistas de diversas instituciones para compartir avances teóricos, metodológicos y aplicados, fomentando la colaboración científica y el desarrollo de proyectos conjuntos. La relevancia de esta instancia radica en consolidar redes de trabajo que potencien la investigación en teoría fuzzy, favoreciendo su impacto en distintas áreas del conocimiento y en la resolución de problemas complejos en contextos reales.

Organización

Las Jornadas Chilenas de Teoría Fuzzy 2025 son organizadas por el Laboratorio de investigación Lab[e]saM de la Universidad de Playa Ancha, fundado el 20 de junio de 2007 y se formalizó administrativamente en el año 2015 mediante el Decreto Exento N° 4087/2015. El objetivo general del Lab[e]saM, es crear un ambiente propicio para la investigación y difusión de los conocimientos correspondientes. Busca fomentar la discusión científica de excelencia, abierta e inclusiva, con el propósito de alcanzar reconocimiento a nivel regional y nacional. Además, contribuye a la investigación mediante la formación de jóvenes investigadores y busca consolidar un equipo de trabajo activo y dinámico, con un sólido conocimiento de las disciplinas teóricas que respaldan sus actividades.

Comité organizador

Dr. Cristian Carvajal   Dr. Cristian Mejías   Dr. Ronald Manríquez

Programa

CI: Conferencia invitada, EI: Exposiciones de Investigadores e investigadoras.

Jueves 4 de Septiembre

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12:00–14:00	**Almuerzo**		
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14:25–14:45	EI	**Mg. Rocío Cárdenas** Universidad de Tarapacá, Chile	On the best polygonal approximation of fuzzy numbers by using a family of metrics based on mid-spread formulation
14:50–15:10	EI	**Dr. Roberto Díaz** Universidad de los Lagos, Chile	Ley de tricotomía para números difusos triangulares basados en órdenes admisibles
14:50–15:10	EI	**Dr. Nicolás Zumelzu** Universidad de Magallanes, Chile	Fuzzificación de datos y definición de términos lingüísticos
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Viernes, 5 de Septiembre

9:00-9:40	CI	**Dr. Estevão Esmi** Universidades Estadual de Campinas, Brasil	Fuzzy Integral via Interactivity
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10:15-10:35	EI	**Rafael Cordero; Sonia Estay, Yerko Reygadas** Universidad de Playa Ancha, Chile	The fuzzy hyperbola
10:40–11:00	EI	**Dr. Alejandro Velozo** Universidad de Valparaíso, Chile	Fuzzy modeling in Functional Magnetic Resonance Imaging
11:05–11:25	EI	**Marcia Aguilera; Samantha Neira** Universidad de Playa Ancha, Chile	The fuzzy ellipse
11:30–11:50	EI	**Dr. Ronald Manríquez** Universidad de Playa Ancha, Chile	Fuzzy metric space
12:00–14:00	**Almuerzo**		
14:00–14:40	CI	**Dr. Benjamín C. Bedregal** Universidade Federal do Rio Grande do Norte, Brasil	Integrales de Choquet
15:00–15:30	**Cierre de las jornadas y Cóctel**		

Conferencias – Charlas

Conferencias invitadas

Fuzzy Integral via Interactivity

Estevão Esmi **CT**

Universidades Estadual de Campinas, Brasil

Fuzzy integrals are considered well-established by many researchers in the field. The main reason for this is that they possess most of the usual properties of classical integrals, including linearity. Since the early 2000s, the notions of Aumann, Riemann, and Henstock fuzzy integrals have been established in the literature. These three notions of fuzzy integrals coincide whenever the integrand function is continuous. This talk aims to introduce a new approach to fuzzy integral based on the notion of summation of interactive fuzzy numbers. In particular, we introduce the notion of γ-Riemann fuzzy integration, where γ is a parameter in the unit interval that plays the role of controlling the uncertainty of the corresponding integral. The existence of the γ-Riemann integral of a function is connected to the existence of its usual fuzzy Riemann integral. Furthermore, the proposed notion of fuzzy integral has properties similar to those for the fuzzy Riemann integral.

Aggregation Processes by Fuzzy Integrals and their Generalizations

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Special aggregation functions, which have been attracting much attention in the literature, are the Choquet and Sugeno integrals [1, 2], which are fuzzy integrals that can be used to integrate a function with respect to a (non-additive) fuzzy measure [3], allowing to consider the relationship among the attributes been integrated.
In particular, the Choquet integral was introduced in 1953, but it was mentioned for applications only after 1986, when it was used for the development of concepts usually applied in economics, such as decision making based on preference relations [4] in 1986 and expected utility maximization criterion under uncertainty [5] in 1989. Since then, the discrete Choquet integral has been applied in a large number of fields, mainly when it is important to consider the interactions among the elements to be integrated [6], such as in decision making [7, 8] and multi-criteria decision making [9, 10], deep learning [11], image processing [12], risk evaluation [13], classification problems [14, 15] and brain computer interfaces [16, 17].
Generalizations of the Choquet and Sugeno integral started to appear in the literature in 1987,with the works by Sugeno and Murofushi [18], followed by Mesiar and Grabisch et al. [19] in 1995, and, more recently, from 2016 on, by the UPNA-Brazil research group (Dimuro, Bustince, Bedregal, Lucca, Asmus, Takáč, Santos and Wieczynski) [10, 14, 15, 20, 21, 22, 23, 24, 25], and by several other authors, like Zhang et al. (2022) [26] and Boczek et al. (from 2023 on) [27, 28].

In this talk, we present the fundamentals and applications of Choquet and Sugeno integrals and their generalizations, pointing to the future of such kinds of fusion operators.

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Integrales de Choquet

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En 1953, el matemático francés Gustave Choquet desarrolló la teoría de capacidades [1], una generalización de las medidas empleadas por Lebesgue en sus integrales [3], eliminando la aditividad. Con esto, Choquet introdujo y estudió un nuevo tipo de integral, hoy conocida como la integral de Choquet. Esta integral es utilizada para representar la importancia relativa de diferentes grupos de criterios (coaliciones) y fueron aplicadas en campos como la teoría de decisiones y la optimización multiobjetivo para capturar relaciones no convexas. Sin embargo, la versión discreta de la integral de Choquet, es decir, cuando el espacio es finito, solo se ha investigado a partir del trabajo de [5, 6], aunque no de forma explícita. El primer registro que encontré donde se define y se le da el nombre de integral de Choquet discreta fue [2], pero fue Peter Denneberg quien la popularizó al conectarla con la teoría de conjuntos difusos y el procesamiento de la información. En este contexto, estas integrales son una clase importante de funciones de agregación de tipo medio.
No tardó mucho en surgir generalizaciones de la integral de Choquet, y en particular de la integral de Choquet discreta. Estas incluyen la generalización de la t-conorma en [5] y la de Radko Mesiar en [4], quien utiliza la pseudomultiplicación y la pseudoadición para definir una generalización de las integrales de Choquet que denominó integrales de tipo Choquet. Esta expresión se utiliza actualmente para variantes de las integrales de Choquet discretas.
En esta presentación, analizaremos las integrales de Choquet discretas, formas alternativas de definición y una caracterización en función de las propiedades que cumplen. Analizaremos algunas variantes de la integral de Choquet y los tipos de problemas a los que se han aplicado.

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A new family of admissible orders on fuzzy numbers

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In this work, we introduce a new family of admissible orders on fuzzy numbers, based on the mid/spread representation. This formulation refines the conventional partial order, as well as other well-established partial orders on fuzzy numbers.

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Contribuciones

Fuzzificación de datos y definición de términos lingüísticos

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En esta exposición se introducirá, en primer lugar, la fuzzificación de datos, proceso que permite transformar valores numéricos en representaciones fuzzy mediante los llamados números fuzzy [1], [3], [5] y [4]. Posteriormente, se abordarán los órdenes admisibles sobre números fuzzy (ver [1]), una herramienta matemática clave para extender conceptos clásicos de comparación y jerarquización al ámbito de los conjuntos fuzzy. Asimismo, se definirán los términos lingüísticos asociados a las variables de estudio ([2]). Finalmente, se presentará la definición de órdenes admisibles, sus propiedades fundamentales y su relevancia en el tratamiento y la jerarquización de dichos términos, con el fin de comprender mejor el comportamiento de las variables, destacando cómo estas estructuras permiten comparar valores fuzzy de manera coherente con las nociones de incertidumbre y aproximación.

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On the best polygonal approximation of fuzzy numbers by using a family of metrics based on mid-spread formulation

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In this work, we study how to approximate a fuzzy number using level-m polygonal fuzzy numbers. To this end, we introduce a general family of metrics d_θ^Ψ, based on the mid-spread representation, which generalizes previous metrics by allowing non-constant weighting functions. This provides the flexibility to emphasize either the position or the spread of the α-cuts, depending on the case. Using a new matrix representation, we show that the best approximation problem can be formulated as a convex quadratic optimization problem with linear constraints in a finite-dimensional space. We illustrate the method with the case of the best trapezoidal approximation

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Aplicación de la Optimización Difusa en la solución de problemas de Scheduling Machine con imprecisión en la realización de las tareas

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En los problemas de Scheduling Machine, las tareas compiten por utilizar las máquinas, pero el tiempo de ejecución de las tareas puede ser immpreciso, ya que puede haber retrazos en el inicio o en el tiempo de ejecución de las tareas; si la imprecisión no se considera en el modelo, los planes obtenidos para la realización de las tareas puede resultar inválido. En este trabajo se muestra aplicaciónes de la optimización difusa para resolver problemas de scheduling machine donde ocurren problemas de imprecisión en la realización de las tareas como el sector construcción, atraque de Barcos y máquinas en paralelo.

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The fuzzy Ellipse

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The theory of fuzzy analytic geometry, formally introduced by Buckley and Eslami [1], provides a rigorous framework for studying geometric figures under uncertainty. Since then, significant advances have been made by Ghosh, Chakraborty, and collaborators [2] who extended the field with definitions of points, line segments, distances, lines, triangles, circles, and tangents. Further developments include studies on metric relations in fuzzy right triangles [3], tangents to fuzzy circles [4], and recent guidelines for fuzzy spatial analytic geometry [2]. Earlier contributions by Rosenfeld (1984), motivated by image processing and pattern recognition, introduced notions such as fuzzy height, width, and diameter, which later inspired research on fuzzy geometry and topology of subsets, including adjacency, separation, and connectivity. In this work, we focus on the definition of the fuzzy ellipse. By employing α-cut sets and modeling the foci as fuzzy points, the fuzzy ellipse is interpreted as a family of classical ellipses whose foci lie on the supports of these fuzzy points. Graphical representations and simulations illustrate the construction, providing an intuitive understanding of its geometry. Additionally, we explore related concepts such as fuzzy center, fuzzy vertex, and fuzzy eccentricity, contributing to the ongoing development of fuzzy conics in analytic geometry.

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Fuzzy modeling in Functional Magnetic Resonance Imaging

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This work introduces a comprehensive pipeline for functional magnetic resonance imaging (fMRI) analysis that incorporates fuzzy modeling to address the intrinsic variability and uncertainty of the hemodynamic response function (HRF) across brain regions, subjects, and time. The core contribution is the definition of a fuzzy HRF, utilized within a fuzzy general linear model (GLM) framework and a fuzzy deconvolution methodology for robust estimation of neural events. The fuzzy GLM, which can be practically implemented using either LR fuzzy number representation or the more general α-cut approach, enhances the capture of intra- and intersubject HR variability, supports the computation of activation maps, and enables rigorous statistical power assessment. Furthermore, a novel fuzzy vector autoregressive (FVAR) model is proposed for brain connectivity analysis, using fuzzy numbers to aggregate ROI signals and improve network reliability through regularization and sparsity promotion. Evaluations with in silico and in vivo fMRI data demonstrate that these fuzzy approaches yield robust estimation of neural events, reduce false negatives in activation mapping, and outperform conventional methods in connectivity identification, showing particularly promising improvements in clinical populations such as individuals with autism spectrum disorders. The methodology consistently achieves superior accuracy and reliability in both activity and connectivity analyses, underscoring the advantages of fuzzy models over fixed HRF assumptions and standard averaging techniques. Future work may include more flexible fuzzy number representations, extended validation on diverse fMRI datasets, and enhanced modeling for complex brain pathologies.

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De la Salud Pública a los Polímeros: Sistemas Fuzzy en Acción

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Los sistemas fuzzy y neuro-fuzzy ofrecen herramientas potentes para abordar problemas que requieren decisiones con información imprecisa o subjetiva. En salud pública, se aplicó un sistema fuzzy basado en reglas para analizar retrospectivamente la implementación del Plan Paso a Paso en Valdivia, mostrando que la incorporación de inteligencia artificial puede mejorar la transparencia y la consistencia en la toma de decisiones durante emergencias sanitarias. De manera análoga, en el ámbito de materiales, se empleó un sistema ANFIS para automatizar la identificación del tramo elástico en polímeros impresos en 3D y calcular el módulo de Young, reduciendo la subjetividad del operador y aumentando la consistencia de los análisis experimentales. Estos ejemplos destacan cómo la Teoría de Conjuntos Fuzzy puede integrarse en contextos distintos para mejorar la objetividad y confiabilidad de decisiones y mediciones.

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The fuzzy hyperbola

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Fuzzy analytic geometry was formally introduced by researchers Buckley and Eslami in the paper [1]. Subsequently, several rigorous papers, led by Gosh and Chakraborty and their collaborators, have been developing fuzzy geometric ideas, such as point, segment, line, triangle, circle, and distance, both in the plane and in space (see [2] and references therein) and recently, the study of metric relations in the fuzzy right triangle and the definition of tangent to a fuzzy circle (see [3] and [4]). However, fuzzy geometric ideas appear much earlier than Buckley and Eslami's proposal. As early as 1984, Rosenfeld motivated by the development of image processing and pattern recognition, introduced ideas of fuzzy height, width, and diameter [5], but without a clearly established mathematical framework. Since then, there has been a growing body of work on fuzzy geometry and the topology of fuzzy image subsets, including adjacency, separation, and connectivity, see for instance [6]. Other geometric ideas have been proposed, such as fuzzy triangles, fuzzy trigonometric functions, areas and perimeter of fuzzy regions, fuzzy lines, and fuzzy line segments; see, for instance, [7]. In the framework of fuzzy geometry, we address the definition of a fuzzy hyperbola in this talk. For the construction of this, the foci are considered fuzzy points and then the fuzzy hyperbola is seen as a collection of crisp hyperbolas with foci at the support of each fuzzy point. The proposal is supported with good graphics that illustrate the fuzzy hyperbola in a good way.

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Ley de tricotomía para números difusos triangulares basados en órdenes admisibles

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La ley de tricotomía para números reales desempeña un papel importante en el análisis real. Sin embargo, existen diversas versiones de esta ley que, al adaptarse al contexto de los números difusos, dependiendo del orden asociado, pueden no ser equivalentes. En este trabajo, consideraremos la clase de números difusos triangulares (NFT's) dotados de un orden admisible e investigaremos las condiciones necesarias (y suficientes en algunos casos) que dicho orden admisible debe cumplir para satisfacer ciertas versiones de la ley de tricotomía para NFT's.

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Fuzzy metric space

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Fuzzy metric spaces are an extension of classical metric spaces in the context of fuzzy sets, and since the late 1970s, various authors have introduced the concept of a fuzzy metric space in different forms, see, for example, [1], [2], and [3]. These ideas are used in functional analysis, general topology, control theory, fuzzy neural networks, and approximation theory. However, none of them considers the natural extension of the metric axioms to the fuzzy frame, i.e., that the distance between two objects must be a fuzzy number and that it satisfies the metric axioms, that is, non-negativity, identity of indiscernibles, symmetry, and triangle inequality. In this talk, we will show some definitions of fuzzy metric spaces, and we will show a different proposal from the previous authors, illustrated with a case of fuzzy geometry.

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